Eggleston meets Mycielski - category case

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Let us recall the following the two following theorems on inscribing special kind rectangles and squares into large subsets of the plane.

Theorem 1 (Eggleston [1])**.** *For every conull set $F \subseteq [0,1]^2$ there are a perfect set $P \subseteq [0,1]$ and conull $B \subseteq [0,1]$ such that $P \times B \subseteq F$.*

Theorem 2 (Mycielski [4])**.** *For every comeager or conull set $X \subseteq [0,1]^2$ there exists a perfect set $P \subseteq [0,1]$ such that $P \times P \subseteq X \cup \Delta$, where $\Delta = \{(x,x) : x \in [0,1]\}$.*

We will consider the category variant of the former (comeager instead of conull) in the Cantor space 2^ω and its strengthening via replacing a perfect set with a body of some type of a perfect tree. Mainly we will focus on uniformly perfect trees, Silver trees and Spinas trees. Moreover we will explore the possibility of conjoining the above theorems by demanding that for a comeager set $G \subseteq 2^\omega \times 2^\omega$ there is a comeager set $B \subseteq 2^\omega$ and a tree T of certain kind such that $[T] \times B \subseteq G$ (modulo diagonal) and $[T] \subseteq B$.

The results were obtained together with Robert Rałowski and Szymon Żeberski and and can be found in [2] and [3].

References

[1] Eggleston, H. G., *Two measure properties of Cartesian product sets*, The Quarterly Journal of Mathematics 5, pp. 108–115 (1954).

[2] Michalski M., Rałowski R., Żeberski Sz., *Around Eggleston theorem*, arXiv:2307.07020 (2023).

[3] Michalski M., Rałowski R. Żeberski Sz., *Mycielski among trees*, Mathematical Logic Quarterly 67 (3), pp. 271–281 (2021).

[4] Mycielski J., *Algebraic independence and measure*, Fundamenta Mathematicae 61, pp. 165–169 (1967).